K25P 1900

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.C.S.S. – OBE-Reg./Supple./Imp.)
Examination, April 2025
(2023 and 2024 Admissions)
MATHEMATICS
MSMAT02C08 : Advanced Real Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 5** questions from this Part. **Each** question carries **4** marks.

1. Define a complete metric space. Give an example.
2. State Stone's generalization of the Weierstrass theorem.
3. Define a trigonometric polynomial and prove that every trigonometric polynomial is periodic.
4. For the gamma function prove that $\log \Gamma$ is convex on $(0,\infty)$.
5. State inverse function theorem.
6. Suppose E is an open set in $\mathbb{R}^n$, f maps E into $\mathbb{R}^m$ and $x \in E$. Define the derivative of f at x. **(5×4=20)**

PART – B

Answer **any 3** questions from this Part. **Each** question carries **7** marks.

7. Suppose $\lim_{n \to \infty} f_n(x) = f(x)$, $x \in E$ and put $M_n = \sup |f_n(x) - f(x)|$. Prove that $f_n \to f$ uniformly on E if and only if $M_n \to 0$ as $n \to \infty$.
8. Prove : A sequence $\{f_n\}$ converges to f with respect to the metric of $\mathscr{C}(X)$ if and only if $f_n \to f$ uniformly on X.
9. Let $f_n(x) = \sin nx$ where $0 \le x \le 2\pi$, n = 1, 2, 3,... Prove that there does not exists a subsequence $\left\{f_{n_k}\right\}$ of $\{f_n\}$ which converges pointwise on $[0, 2\pi]$.

10. Let $\{\phi_n\}$ be orthonormal on [a, b]. Let $s_n(x) = \sum_{m=1}^{n} C_m \phi_m(x)$ be the n^{th} partial sum of the Fourier series of f and suppose $f_n(x) = \sum_{m=1}^{n} \gamma_m \phi_m(x)$. Then prove that $\int_a^b |f - s_n|^2 dx \leq \int_a^b |f - t_n|^2 dx$ and equality holds if and only if $\gamma_m = C_m$ for m = 1, 2, 3, ..., n.

11. Prove : Suppose f maps an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$ and f is differentiable at a point $x \in E$. Then the partial derivatives $(D_j f_j)$ (x) exist and $f'(x)e_j = \sum_{i=1}^{m} (D_j f_j)(x) u_i$ $(1 \leq j \leq n)$ where $e_1, e_2, ..., e_n$ and $u_1, u_2, ..., u_m$ are standard bases for $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively. **(3×7=21)**

PART – C

Answer **any 3** questions from this Part. **Each** question carries **13** marks.

12. Let X be a metric space and $\mathscr{C}(X)$ be the set of all complex valued continuous bounded functions with domain X. Prove that $\mathscr{C}(X)$ is complete metric space.

13. a) Define pointwise bounded sequence of functions.

    b) Let α be monotonically increasing on [a, b]. Suppose $f_n \in \mathscr{R}(\alpha)$ on [a, b], for n = 1, 2, 3, ... and suppose $f_n \to f$ uniformly on [a, b]. Then prove that $f \in \mathscr{R}(\alpha)$ on [a, b] and $\int_a^b f dx = \lim_{n\to\infty} \int_a^b f_n dx$.

14. State and prove Weierstrass theorem.

15. Suppose $a_0, a_1, a_2, ..., a_n$ are complex numbers, $n \geq 1$. $a_n \neq 0$, $P(z) = \sum_0^n a_k z^k$. Then prove that P(z) = 0 for some complex number z.

16. State and prove implicit function theorem. **(3×13=39)**